Engineering Notes

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Estimation of Taxi Load Exceedances Using Power Spectral Methods

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THE use of power spectral methods in the analysis of repeated and severe ground loads induced on airplanes by runway or taxiway roughness has considerably lagged behind similar utilizations in the analysis of gust loadings. Specific technical problem areas which have inhibited this utilization include 1) the lack of sufficient statistical data in power spectral form defining the operational runway and taxiway roughness environment, 2) the consideration that isolated bumps and dips of a discrete nature in runways tend to invalidate the concept of a stationary Gaussian process, 3) the lack of supporting statistical response data to verify the symmetric or antisymmetric load responses to be considered, and 4) the difficulty of concisely representing nonlinear landing gear characteristics.

Each one of these technical areas in question with regard to ground loads has a counterpart with regard to gust loads. In the latter case, however, power spectral analysis techniques have proved to be extremely useful in design studies despite the lack of rigorous resolution of problem areas similar to those mentioned previously. The purpose of this Note is to present a few key results of taxi loads developmental work funded internally at Lockheed-Georgia which indicate that power spectral techniques applied to the airplane taxiing problem will yield results that are valid to approximately the same degree as similar results for the gust problem.

The first step in the formulation of power spectral taxi loads criteria was taken in the direction of updating the description of the environment in terms of the power spectral density of the surface roughness using data extracted from Ref. 1 and the supporting documents referred to therein. The measured data for runways of 5000 ft or longer (32 U.S. Air Force runways and 32 NATO nation or other runways) were used to establish the environment on the basis that these data are representative of currently operational runways. The surfaces were surveyed along the runway centerline and, in many cases, along other lines parallel to the centerline so that a total of 115 power spectral roughness data sets were included in the data sample. From these data, new roughness criteria were formulated within a framework that provided for an interpretation of the roughness environment in discrete levels of severity so that nonlinear airplane-gear responses as influenced by changes in over-all roughness levels could be utilized if required. At the same time, it was desired to generalize and simplify these criteria as much as practical without compromising the over-all accuracy of the analysis results.

At each of seven different roughness wavelengths selected in the analysis, the cumulative probability of exceeding a given power spectral roughness density was plotted, faired, and subdivided into four selected roughness ranges, each of which represented a given fraction of the 115 runway surfaces surveyed as indicated by the data points plotted in Fig. 1. A separate but similar analysis was also made for taxiway roughness using spectral densities from Ref. 1 based on 56 surveys of 21 U.S. Air Force taxiways. These results are also shown in Fig. 1.

The four discrete roughness levels finally established were aimed at a simplified criterion for analysis purposes using a cut-and-try process to assure compatibility of roughness level and probability of occurrence. It was found that by maintaining a constant spectral shape and allowing the roughness levels to vary in multiples of 2.0, a satisfactory spread of roughnesses and exposure probabilities was obtained. To effect a simplified criterion for roughness, the data at $\Omega = 5.0$ rad/ft were ignored since the roughness at this wavelength tends to be absorbed by the tires and bogies of large airplanes. The data at $\Omega = 0.02$ and $\Omega = 0.01$ rad/ft were also partially disregarded in a conservative manner in order to maintain a consistent family of curves. The resulting set of roughness spectral densities are described by

$$\Phi(\Omega)_n = 2^{n-1}(5.0 \times 10^{-9}\Omega^{-5.35} + 6.25 \times 10^{-4}\Omega^{-2.10}) \quad (1)$$

where $\Phi(\Omega)_n$ is the power spectral density of the nth surface in inches squared per rad/ft and n is the runway surface 1, 2, 3, or 4. The taxiway data were not treated separately since speeds on the taxiway are low and structural responses in a spectral representation would be similar to responses on the runway.

The first term on the right-hand side of Eq. (1) represents long wavelength roughness which is not accurately known, and in most cases the significance of this response to the total

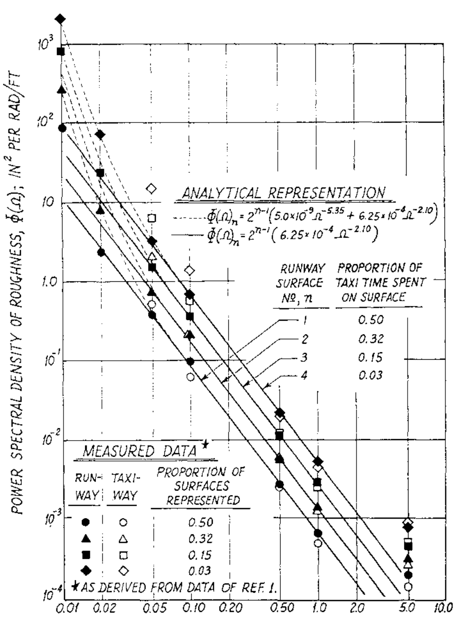

Fig. 1 Power spectral runway roughness criteria compared with measured data.

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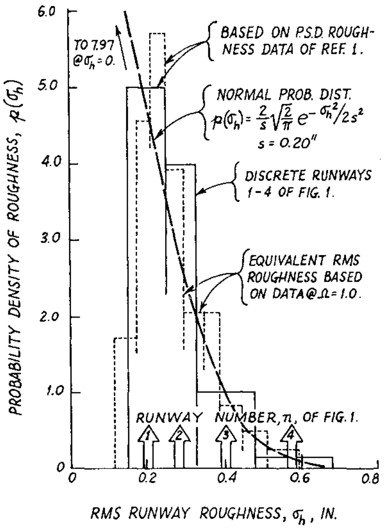

Fig. 2 Probability density distributions of rms runway roughness height.

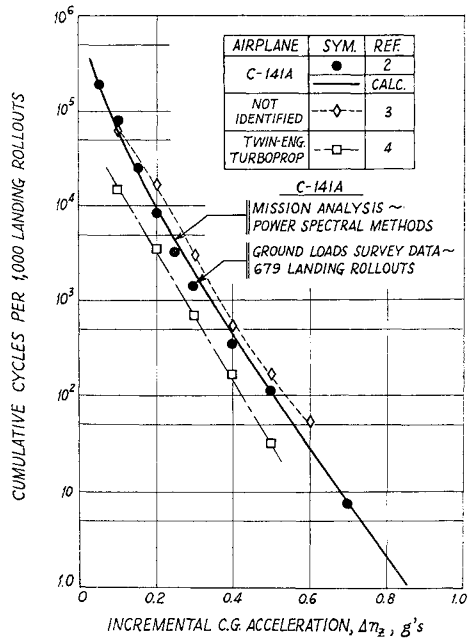

Fig. 3 Comparison of C-141A landing rollout c.g. acceleration spectrum with measured data.

load exceedance spectrum is small. Therefore, current taxi studies at Lockheed-Georgia do not include the effects of this first term in structural response calculations.

Figure 2 shows the probability distribution of runway roughness height h expressed in terms of the rms roughness of the runway σ_h given here by the square root of the integral of the spectral density from $\Omega = 0.02$ rad/ft to ∞. Using a normal distribution as in gust analyses to fit these data, the variance of the distribution is $s^2 = 0.04$ in.^{2} with a scale factor of 2.0 applied to all ordinates of the normal curve. This distribution is plotted in Fig. 2 which illustrates the incompatibility of the normal curve with the measured data at the low roughness levels. This same discrepancy has been noted previously in the gust representations as well, but it is known to have negligible effect on the total repeated loads spectrum.

The analogy to the gust case may be carried further by assuming a linear system response and constant speed conditions using Rice's approximation for exceedances of the response variable y for a time period T given by

$$N(y) = N_0 T \exp(-y^2/2\sigma_y^2) \tag{2}$$

where N_0 and σ_y are the characteristic frequency and rms output response of y, respectively, as determined from output/input relations analogous to those used in gust analyses. If the normal distribution of rms runway roughnesses of Fig. 2 is used, the total exceedances of the response variable during exposure to all roughness levels may be expressed as

$$N(y)_S = N_0 T \int_0^\infty \frac{2}{s}\left(\frac{2}{\pi}\right)^{1/2} \exp\left(-\frac{\sigma_h^2}{2s^2} - \frac{y^2}{2\sigma_y^2}\right) d\sigma_h \tag{3}$$

where the subscript S denotes total exceedances from all runway surfaces. Using the relation $R = \sigma_y/\sigma_h$, Eq. (3) may be integrated to yield

$$N(y)_S = 2N_0 T \exp(-y/sR) \tag{4}$$

which is similar to an equivalent expression for gust exceedances. A solution for total exceedances using the four discrete runways with Eq. (2) was compared with Eq. (4) which showed that Eq. (4) was a good approximation to the summation of exceedances on the four runways of Fig. 1.

Tests of Spectral Taxi Loads Criteria

Dynamic response test data from the C-130 and C-141A aircraft during constant speed taxi runs have verified the applicability of Eq. (2) for a number of response variables, including the acceleration of the fuselage at a point corresponding to the static c.g. position. Analog and digital program studies of taxiing response of these two aircraft have also shown that an equivalent linear system representation over most of the roughness/speed range in normal operations is a reasonable approach for load exceedance purposes to the extent that the characteristic frequency and rms output response quantities may be predicted with reasonable accuracy from a known input roughness spectrum and linearized response. Since a large amount of ground loads survey data on the C-141A were included in a structural loads program[2] sponsored by the U.S. Air Force, data for this airplane were used in a preliminary linearized analysis as a test of the spectral representation of taxi loads as outlined here.

The steps in this analysis include the subdivision of operational experiences according to type of event such as takeoff roll, landing rollout, etc., and further subdivision of each event into gross weight-speed segments along with the elapsed time associated with each mission segment. The landing rollout spectrum was selected for analysis purposes since it is also typical of load factor spectra reported[2] for the touch-and-go and the takeoff roll and is equal to or more severe than load factor spectra from these ground events. Six constant-speed taxi cases were used for each of three representative gross weights chosen for the analysis, where the time spent in each mission segment was determined from rational considerations of utilization data and the velocity profile following touchdown.

Data for the rms c.g. acceleration response ratio R and N_0 were computed for each mission segment using a linearized gear representation in a symmetric response analysis which included airplane structural flexibility effects as well as the appropriate time lags between nose and main gear excitation. Aerodynamic effects were not included because spoilers on the C-141A are deployed immediately after touchdown, resulting in the loss of all significant wing lift. Values of R and N_0 for each mission segment were used in Eq. (4) with $s = 0.20$ in. from Fig. 2 and summed for all mission segments. The results of the analysis shown in Fig. 3 compare very favorably with the C-141A test data of Ref. 2 indicating that this approach is satisfactory for the symmetric taxi events. To illustrate that the C-141A data are not atypical, some data from Refs. 3 and 4 are included for comparison purposes.

References

[1] Hahn, E. E., "Design Criteria for Ground-Induced Dynamic Loads," RTD-TDR-63-4139, Nov. 1963, Research & Technology Div., Wright-Patterson Air Force Base, Ohio.

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[3] Taylor, J., *Manual on Aircraft Loads*, 1st ed., Pergamon, New York, 1965, p. 153.

[4] Hunter, P. A., "An Analysis of VG and VGH Operational Data From a Twin-Engine Turboprop Transport Airplane," TN D-1925, July 1963, NASA.

A Convenient, Explicit Formula for Oblique-Shock Calculations

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THIS Note presents an approximate formula enabling direct calculation of changes in flow properties across an oblique shock wave, which has been found to be quite useful by the author. The need for such an explicit relationship (in terms of the approach Mach number and the deflection angle) has been pointed out many times (cf. Ref. 1) and several approximate methods have been developed to fulfill this need. Among these are the Busemann power series (cf. Ref. 1), the first two terms of which are referred to as the supersonic linear and second-order theory, respectively, and several hypersonic approximations.[2–5] The formula presented in this Note is more accurate than any of these and serves as a good starting point for an iterative solution or as a fairly accurate engineering solution for systems or preliminary design work where an explicit equation is necessary.

By neglecting one term in the exact cubic equation for $\sin^2\theta$, Cleary and Axelson[3] arrived at the following relation:

$$\sin^2\theta = \tfrac{1}{2} + 1/M^2 + \tfrac{1}{2}\gamma \sin^2\delta - \cos\delta[\tfrac{1}{4} - 1/M^2 - \gamma^2 \sin^2\delta/4]^{1/2} \quad (1)$$

where θ is the shock-wave angle and δ is the deflection angle. (All other flow properties may be calculated conveniently from $\sin^2\theta$, cf. Ref. 1.) If the neglected term were retained, we would obtain the following exact (but implicit) equation:

$$\sin^2\theta = \tfrac{1}{2} + 1/M^2 + \tfrac{1}{2}\gamma \sin^2\delta - \cos\delta[\tfrac{1}{4} - 1/M^2 - \gamma^2 \sin^2\delta/4 + 1/(M^4 \sin^2\theta)]^{1/2} \quad (2)$$

where only the final term differs from Eq. (1).

If we look upon Eq. (2) as a successive approximation scheme, we see that the Cleary and Axelson formula [Eq. (1)] amounts to taking $1/M^4 \sin^2\theta = 0$ as a first guess. By substituting a more accurate initial guess, we can obtain a much better approximation. For this purpose, the modified hypersonic approximation of Bertram and Cook was selected. This approximation was found to be generally more accurate than other existing approximations, such as the modified hypersonic equation of Van Dyke[5] or the first- or second-order supersonic theory. Bertram and Cook's approximation can be written as follows:

$$\sin^2\theta = \left\{\frac{\gamma+1}{4}\frac{M}{\beta}\sin\delta + \left[\frac{1}{M^2} + \left(\frac{\gamma+1}{4}\frac{M}{\beta}\sin\delta\right)^2\right]^{1/2}\right\}^2 \quad (3)$$

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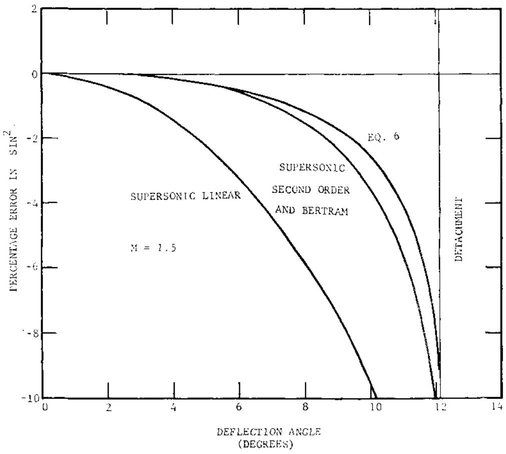
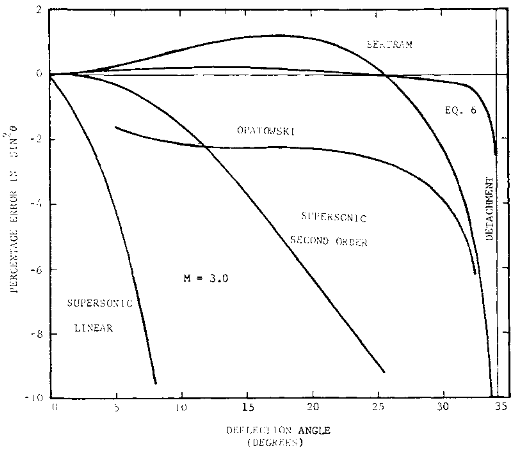


a) M = 1.5



b) M = 3.0

Fig. 1 Accuracy of various approximations vs deflection angle.

where $\beta = (M^2 - 1)^{1/2}$. Substituting this into the last term of Eq. (2), that term becomes

$$\frac{1}{(M^4 \sin^2\theta)} = M^{-2}\left\{\frac{\gamma+1}{4}\frac{M^2}{\beta}\sin\delta + \left[1 + \left(\frac{\gamma+1}{4}\frac{M^2}{\beta}\right)^2\right]^{1/2}\right\}^{-2} \quad (4)$$

For ease in writing the final equation, and to facilitate hand calculations, we define

$$A \equiv [(\gamma+1)/4](M^2/\beta)\sin\delta \quad (5)$$

Equation (2) now becomes

$$\sin^2\theta = \tfrac{1}{2} + 1/M^2 + \tfrac{1}{2}\gamma\sin^2\delta - \cos\delta\{\tfrac{1}{4} - 1/M^2 - \gamma^2\sin^2\delta/4 + M^{-2}[A + (1 + A^2)^{1/2}]^{-2}\}^{1/2} \quad (6)$$

which is the desired approximate formula.

The accuracy of this approximation is compared to that of several other approximations in Figs. 1 and 2. In these figures, the supersonic linear and second-order theories for $\sin^2\theta$ were obtained by applying the exact relationship between $\sin^2\theta$ and pressure ratio to the series for pressure ratio given in Ref. 1. The percentage error in pressure ratio is almost identical to that in $\sin^2\theta$. The error in pressure coefficient varies with Mach number and deflection angle, i.e., at low pressure ratios the percentage error in C_p can be several times the error in pressure ratio. The relative accuracy of the various methods is the same, however.